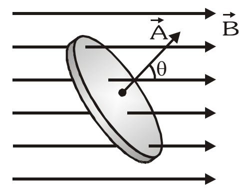
ELECTROMAGNETIC INDUCTION

MAGNETIC FLUX

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector) then $\phi = \vec{B}.\vec{A}$ = BA $\cos\theta$



FLUX LINKAGE

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is ϕ = BA $\cos\theta$ If the coil has N turns, the total flux linkage ϕ = NBA $\cos\theta$

- Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = B \times \text{area} = \left[\frac{F}{IL}\right][L^2] \quad \because \; B = \frac{F}{IL\sin\theta} \qquad [\because F = B\,I\,L\,\sin\theta]$$

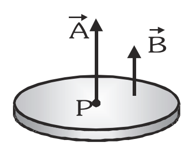
$$\therefore [\phi] = \left[\frac{MLT^2}{AL}\right][L^2] = [ML^2\,T^{-2}\,A^{-1}]$$

SI UNIT of magnetic flux :

$\because$ [M L^2T^{-2}] corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{weber (Wb)} \quad \text{or} \quad T\text{–}m^2 \text{ (as tesla = Wb/m}^2) \qquad \left[\text{ampere} = \frac{\text{coulomb}}{\text{second}}\right]$$

CGS UNIT of magnetic flux : maxwell (Mx) 1Wb = 10^8 Mx



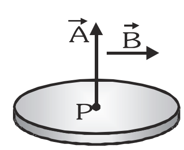
$\phi_{max} = BA$

- For a given area flux will be maximum :

when magnetic field $\vec{B}$ is normal to the area

$\theta = 0 \;\Rightarrow\; \cos\theta$ = maximum = 1 ϕ_{max} = B A

For a given area flux will be minimum :



$\phi_{min} = 0$

when magnetic field $\vec{B}$ is parallel to the area

$\theta = 90 \;\Rightarrow\; \cos\theta$ = minimum = 0 $\qquad \phi_{min} = 0$

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Example

A loop of area $0.06\ m^2$ is placed in a magnetic field 1.2 T with its plane inclined 30 to the field direction. Find the flux linked with plane of loop.

Solution

Area of loop is $0.06\ m^2$, B = 1.2 T and θ = 90 – 30 = 60 So, the flux linked with the loop is

$$\phi = BA\cos\theta = 1.2 \quad 0.06 \quad \cos\theta = 1.2 \quad 0.06 \quad \frac{1}{2} \Rightarrow \phi = 0.036\ Wb$$

Example

A loop of wire is placed in a magnetic field $\vec{B} = 0.3\hat{j}\,T$.

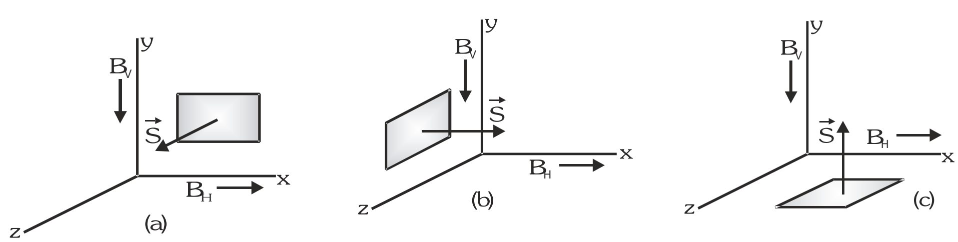
Find the flux through the loop if area vector is $\vec{A} = (2\hat{i} + 5\hat{j} - 3\hat{k})m^2$

Solution

Flux linked with the surface $\phi = \vec{B}.\vec{A} = (0.3\hat{j}).(2\hat{i} + 5\hat{j} - 3\hat{k})m^2$ = 1.5 Wb

Example

At a given plane, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S, if the area S is in (a) x-y plane (b) y-z plane and (c) z-x plane ?



Solution

$\vec{B} = \hat{i}B_H - \hat{j}B_V$ = constant, so $\phi = \vec{B}.\vec{S}$ [$\vec{B}$ = constant]

(a) For area in x-y plane $\vec{S} = S\hat{k}$, $\phi_{xy} = (\hat{i}B_H - \hat{j}B_V).(\hat{k}S) = 0$

(b) For area S in y-z plane $\vec{S} = S\hat{i}$, $\phi_{yz} = (\hat{i}B_H - \hat{j}B_V).(\hat{i}S) = B_H S$

(c) For area S in z-x plane $\vec{S} = S\hat{j}$, $\phi_{zx} = (\hat{i}B_H - \hat{j}B_V).(\hat{j}S) = -B_V S$

Negative sign implies that flux is directed vertically downwards.

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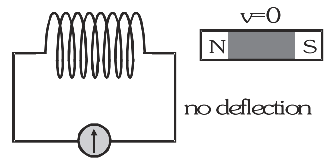
ELECTROMAGNETIC INDUCTION

Michael Faraday demonstrated the reverse effect of Oersted experiment. He explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction. The discovery of this phenomenon brought about a revolution in the field of electric power generation.

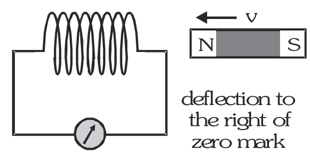
FARADAY'S EXPERIMENT

Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are :

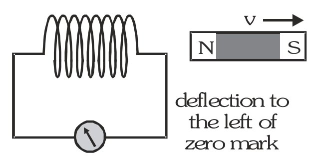
- When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.



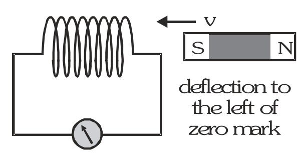
- When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.



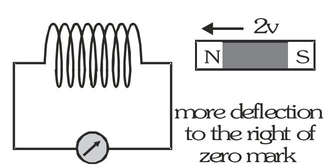
- When the N-pole of a strong bar magnet is moved away from the coil, the galvanometer shows a deflection left to the zero mark.



- If the above experiments are repeated by bringing the S-pole of the, magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.



- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slow.



CONCLUSIONS

Whenever there is a relative motion between the source of magnetic field (magnet) and the coil, an emf is induced in the coil. When the magnet and coil move towards each other then the flux linked with the coil increases and emf is induced. When the magnet and coil move away from each other the magnetic flux linked with the coil decreases, again an emf is induced. This emf lasts so long the flux is changing.

Due to this emf an electric current start to flow and the galvanometer shows deflection.

The deflection in galvanometer last as long the relative motion between the magnet and coil continues.

Whenever relative motion between coil and magnet takes place an induced emf produced in coil. If coil is in closed circuit then current and charge is also induced in the circuit. This phenomenon is called electro magnetic induction.

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FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Based on his experimental studies on the phenomenon of electromagnetic induction, Faraday proposed the following two laws.

- **First law**

Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

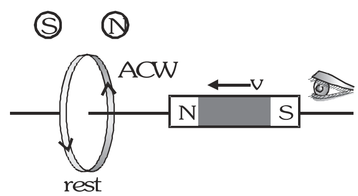
- **Second law**

The magnitude of emf induced in a closed circuit is directly proportional to rate of change of magnetic flux linked with the circuit. If the change in magnetic flux in a time dt is = $d\phi$ then $e \propto \frac{d\phi}{dt}$

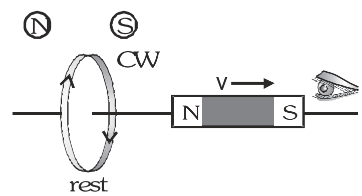
LENZ'S LAW

The Russian scientist H.F. Lenz in 1835 discovered a simple law giving the direction of the induced current produced in a circuit. Lenz's law states that the induced current produced in a circuit always flow in such a direction that it opposes the change or cause that produced it. If the coil has N number of turns and ϕ is the magnetic flux linked with each turn of the coil then, the total magnetic flux linked with the coil at any time = $N\phi$

$$\therefore\ e = -\frac{d}{dt}(N\phi) = -N\frac{d\phi}{dt} = -\frac{N(\phi_2 - \phi_1)}{t}$$



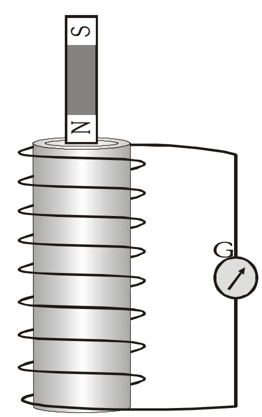
(Coil face behaves as North pole to opposes the motion of magnet.)



(Coil face behaves as South pole to opposes the motion of magnet.)

$e = (-)\frac{d\phi}{dt}$, here negative sign indicates the concept of Lenz law.

LENS'S LAW - A CONSEQUENCE OF CONSERVATION OF ENERGY



Copper coils are wound on a cylindrical cardboard and the two ends of the coil are connected to a sensitive galvanometer. When a bar magnet is moved towards the coil (fig.). The upper face of the coil near the magnet acquired north polarity.

Consequently work has to be done to move the magnet further agains the force of repulsion. When we withdraw the magnet away from the coil, its nearer face acquires south polarity. Now the workdone is against the force of attraction.When the magnet is moved, the number of magnetic lines of force linking the coil changes, which causes an induced current of flow through the coil. The direction of the induced current, according to Lenz's law is always to oppose the motion of the magent.

The workdone in moving the magnet is converted into electrical energy. This energy is dissipated as heat energy in the coil. Therefore, the induced current always flows in such a direction to oppose the cause. Thus it is proved that Lenz's law is the consequence of conservation of energy.

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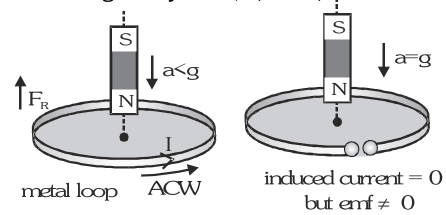
GOLDEN KEY POINTS

- Induced emf does not depends on nature of the coil and its resistance.
- Magnitude of induced emf is directly proportional to the relative speed of coil magnet system, ($e \propto v$).
- Induced current is also depends on resistance of coil (or circuit).
- Induced emf does not depends on resistance of circuit, It exist in open circuit also.



- In all E.M.I. phenomenon, induced emf is non-zero induced parameter.
- Induced charge in any coil (or circuit) does not depends on time in which change in flux occurs i.e. it is independent from rate of change of flux or relative speed of coil–magnet system.
- Induced charge depends on change in flux through the coil and nature of the coil (or circuit) i.e. resistance.

Example

The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m^2 acts perpendicular to the plane of the coil. What will be the radius of the coil when the induced e.m.f. in the 1μV

Solution

Induced emf $e = \dfrac{d(BA)}{dt} = \dfrac{Bd(\pi r^2)}{dt} = 2B\pi r\dfrac{dr}{dt}$ radius of coil $r = \dfrac{e}{2B\pi\left(\dfrac{dr}{dt}\right)} = \dfrac{1\times10^{-6}}{2\times10^{-3}\times\pi\times10^{-2}} = \dfrac{5}{\pi}$ cm

Example

The ends of a search coil having 20 turns, area of cross-section 1 cm^2 and resistance 2 ohms are connected to a ballistic galvanometer of resistance 40 ohms. If the plane of search coil is inclined at 30 to the direction of a magnetic field of intensity 1.5 Wb/m^2, coil is quickly pulled out of the field to a region of zero magnetic field, calculate the charge passed through the galvanometer.

Solution

The total flux linked with the coil having turns N and area A is

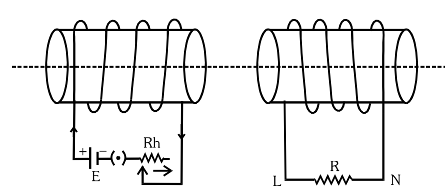
$$\phi_1 = N(\vec{B}.\vec{A}) = NBA\ \cos\theta = NBA\ \cos(90 - 30) = \frac{NBA}{2}$$

when the coil is pulled out, the flux becomes zero, $\phi_2 = 0$ so change in flux is $\Delta\phi = \dfrac{NBA}{2}$

the charge flowed through the circuit is $q = \dfrac{\Delta\phi}{R} = \dfrac{NBA}{2R} = \dfrac{20\times1.5\times10^{-4}}{2\times42} = 0.357\times10^{-4}$ C

Example

When resistance of primary is changed according to figure then what is the direction of induced current in resistance 'R' of secondary?



Solution

L to N]

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Example

The cross-sectional area of a closely-wound coil having 40 turns in 2.0 cm^2 and its resistance is 16 ohm. The ends of the coil are connected to a B.G. of resistance 24 ohm. Calcualte the charge flowing through the B.G. when the coil is pulled quickly out of a region where field is 2.5 Wb/m^2 to a region of zero magnetic field.

Solution

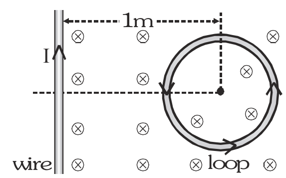
$N = 40$, $B = 2.5$ Wb/m^2 , $A = 2.0$ cm 10^{-4} m^2 and $R = 16 + 24 = 40$ ohms

the charge flowed through the circuit is $q = \frac{NBA}{R} = \frac{40 \times 2.5 \times (2.0 \times 10^{-4})}{40} = 5.0 \times 10^{-4}$ C

Example

A current $i = 3.36(1 + 2t)$ 10^{-2} A increases at a steady rate in a long straight wire. A small circular loop of radius 10^{-3} m has its plane parallel to the wire and its centre is placed at a distance of 1m from the wire. The resistance of the loop is 8.4 10^{-4} Ω. Find the magnitude and the direction of the induced current in the loop.

Solution



The field due to the wire at the centre of loop is $B = \frac{\mu_0 I}{2\pi d} = \times \frac{2 \times 10^{-7} \times I}{1}$

So the flux linked with the loop wire

$\phi = BA = B \quad \pi r^2 = 2\ I \quad 10^{-7} \quad \pi \quad (10^{-3})^2 = 2\pi I \quad 10^{-13}$

So emf induced in the loop due to change of current $|e| = \frac{d\phi}{dt} = 2\pi \times 10^{-13} \frac{dI}{dt}$

$\because\ I = 3.36\ (1 + 2t) \quad 10^{-2} \qquad \therefore\ \frac{dI}{dt} = 6.72 \times 10^{-2}$ A/s

And hence $e = 2\pi \quad 10^{-13} \quad 6.72 \quad 10^{-2} = 13.44\pi \quad 10^{-15}$ V

And the induced current in the loop $i = \frac{e}{R} = \frac{13.44\pi \times 10^{-15}}{8.4 \times 10^{-4}} = 16\pi \times 10^{-12}$ A

Due to increase in current in the wire the flux linked with the loop will increase, so in accordance with Lenz's law the direction of current induced in the loop will be inverse of that in wire, i.e., anticlockwise.

Example

A 10 ohm resistance coil has 1000 turns. It is placed in magnetic field of induction 5 10^{-4} tesla for 0.1 sec. If the area of cross-section is 1m^2, then calculate the induced emf.

Solution

Magnetic flux through the coil $\phi = NBA$ Induced emf $= \frac{d\phi}{dt} = \frac{NBA}{t} = \frac{1000 \times 5 \times 10^{-4} \times 1}{0.1} = 5V$

Example

A coil of mean area 500 cm^2 and having 1000 turns is held perpendicular to a uniform field of 0.4 gauss. the coil is turned through 180 in 1/10 second. Calculate the average induced emf.

Solution

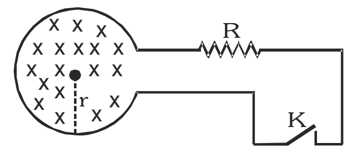
When the plane of coil is perpendicular to field, the angle between area $\vec{A}$ and field $\vec{B}$ is 0 .

The flux linked with coil ϕ_1 = NBA cos 0 = NBA. When coil is turned through 180 , the flux linked ϕ_2 = NBA cos π = – NBA $\therefore$ change in flux $\phi = \phi_2 - \phi_1 = -$ 2NBA

the magnitude of the induced emf is $\varepsilon = -\frac{d\phi}{dt} = \frac{2NBA}{dt} = \frac{2\times1000\times0.4\times10^{-4}\times500\times10^{-4}}{0.1} = 0.04$ V

Example

Shown in the figure is a circular loop of radius r and resistance R. A variable magnetic field of induction $B = B_0\,e^{-t}$ is established inside the coil. If the key (K) is closed. Then calculate the electrical power developed right after closing the key.



Solution

Induced emf $e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt} = \pi r^2 B_0 \frac{d}{dt}(e^{-t}) = -\pi r^2 B_0\, e^{-t}$

At t = 0, $\quad e_0 = B_0\, e^{-0}\,.\,\pi r^2 = B_0\,\pi r^2$

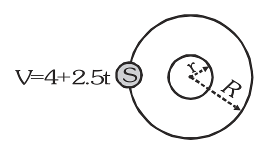
The electric power developed in the resistor R just at the instant of closing the key is $P = \frac{e_0^2}{R} = \frac{B_0^2\pi^2 r^4}{R}$

Example

Two concentric coplanar circular loops made of wire, resistance per unit length $10^{-4}\ \Omega m^{-1}$, have diameters 0.2 m and 2 m. A time-varying potential difference (4 + 2.5 t) volt is applied to the larger loop. Calculate the current in the smaller loop.

Solution

The magnetic field at the centre O due to the current in the larger loop is $B = \frac{\mu_0 I}{2R}$



If ρ is the resistance per unit length, then $I = \frac{\text{potential difference}}{\text{resistance}} = \frac{4+2.5t}{2\pi R\cdot\rho}$ $\quad\therefore B = \frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}$

$\because$ r << R, so the field B can be taken almost constant over the entire area of the smaller loop.

$\therefore$ the flux linked with the smaller loop is $\phi = B\times\pi r^2 = \frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}\cdot\pi r^2$ Induced emf e = $\frac{d\phi}{dt} = \frac{\mu_0 r^2}{4R^2\rho}\times2.5$

The corresponding current in the smaller loop is I' then

$$I' = \frac{e}{R} = \frac{\mu_0 r^2}{4R^2\rho}\times2.5\times\frac{1}{2\pi r\rho} = \frac{2.5\,\mu_0 r}{8\pi R^2\rho^2} = \frac{2.5\times4\pi\times10^{-7}\times0.1}{8\pi\times(1)^2\times(10^{-4})^2} = 1.25A$$

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Example

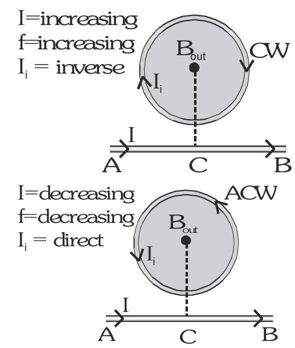
(a) A current from A to B is increasing in magnitude then what is the direction of induced current in the loop and

(b) if current is decreasing in magnitude then what is the direction of current in the loop.

(c) If instead of current if an electron is moving in the same direction, what will happen ?

Solution

(a) When current in the wire AB increases, the flux linked with the loop (which is out of the page) will increase, and hence the induced current in the loop will be inverse, i.e., clockwise and will try to decrease the flux linked with it, i.e., will repel the conductor AB as shown in figure.

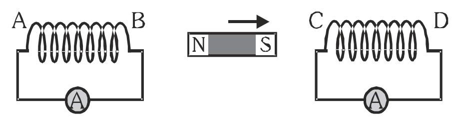


(b) When current in the wire AB decreases, the flux linked with the loop (which is out of the page) will decrease, and hence the induced current in the loop will be in anticlockwise direction and will try to increase the flux linked with it, i.e., will attract the conductor AB as shown in figure.

(c) If an electron moving from left to right, the flux with the loop (which is into the page) will first increase and then decrease as the electron passes by. So the induced current I_i in the loop will be first anticlockwise and will change direction (i.e., will become clockwise) as the electron passes by.

Example

A magnet is moved in the direction indicated by an arrow between two coils AB and CD as shown in the figure. Suggest the direction of induced current in each coil.



Solution

For coil AB : Looking from the end A, the current in the coil AB will be in anticlokwise direction.

For coil CD : Looking from the end D, direction of current in the coil CD will be anticlockwise.

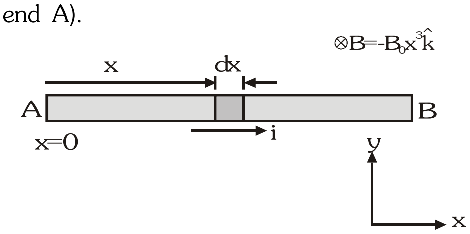
Note : as the N-pole of the magnet is moving away from the coil AB, the end B of the coil will behave as S-pole so as to oppose the motion of the magnet and the end C of the coil CD should behave as S-pole so as to repel the approaching magnet.

Example

In a region of gravity free space, there exists a non-uniform magnetic field $\vec{B} = -B_0 x^3 \tilde{k}$. A uniform conductor AB of length L and mass m is placed with its end A at origin such that it extends along +x-axis. Find the initial acceleration of the centre of mass and that of end A when a current i flows from A to B.

Solution

Consider a small sectionof length dx at a ditance x from left end (or end A).



The force on this section is $d\vec{F} = i\left[dx\tilde{i} \times \left(-B_0 x^3\right)\tilde{k}\right]$ = $B_0 i x^3 dx\tilde{j}$

Force on entire rod is given by,

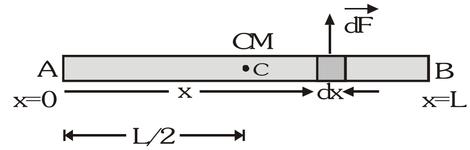
$$\vec{F}_m = \int d\vec{F} = \int_0^L B_0 i x^3 dx\tilde{j} = \frac{B_0 i L^4}{4}\tilde{j} \Rightarrow \vec{a}_{cm} = \frac{\vec{F}_m}{\text{mass}} = \frac{B_0 i L^4}{4m}\tilde{j}$$

To find acceleration of a point on the rod, we first find angular acceleration of the rod about centre of mass. The torque due to $d\vec{F}$ about centre of mass on the rod is

$$d\vec{\tau} = \left(x - \frac{L}{2}\right) i \times d\vec{F} \quad (\because \vec{\tau} = \vec{r} \times \vec{F}) \Rightarrow d\vec{\tau} = \left(x - \frac{L}{2}\right) B_0 i x^3 dx\tilde{k}$$

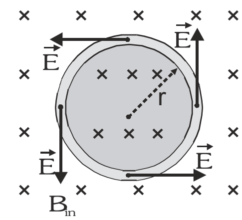
Net torque on the rod is $\tau = \int_0^L B_0 i\tilde{k}\left(x^4 - x^3 \frac{L}{2}\right)dx \Rightarrow \vec{\tau} = B_0 i\tilde{k}\left(\frac{L^5}{5} - \frac{L^5}{8}\right) = \frac{3B_0 i L^5}{40}\tilde{k}$

As $\vec{\tau} = \ell\vec{\alpha}$, $\vec{\alpha} = \frac{\vec{\tau}}{\ell} = \frac{3B_0iL^5}{40\left(\frac{mL^2}{12}\right)}\tilde{k} = \frac{9}{10}\frac{B_0iL^3}{m}\tilde{k}$



Now, acceleration of end A is $\vec{a}_A = \vec{a}_{cm} + \vec{\alpha} \times \vec{r}_{AC}$, where $\vec{r}_{AC}$ is position vector of A with respect to centre of mass $\Rightarrow \vec{a}_A = \frac{B_0iL^4}{4m}\tilde{j} + \left(\frac{9}{10}\frac{B_0iL^3}{m}\tilde{k}\right) \times \left(-\frac{L}{2}\tilde{i}\right)$ $\therefore \vec{a}_A = \frac{B_0iL^4}{4m}\tilde{j} - \frac{9}{20}\frac{B_0iL^4}{m}\tilde{j} = \frac{-B_0iL^4}{5m}\tilde{j}$

INDUCED ELECTRIC FIELD



When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux.

Important properties of induced electric field :

(i) It is non conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf.

Hence $\oint \vec{E}.\vec{d\ell} = e = -\frac{d\phi}{dt}$ ...(i)

This equation is valid only if the path around which we integrate is stationary.

(ii) Due to of symmetry, the electric field $\vec{E}$ has the same magnitude at every point on the circle and it is tangential at each point (figure).

(iii) Being nonconservative field, so the concept of potential has no meaning for such a field.

(iv) This field is different from the conservative electrostatic field produced by stationary charges.

(v) The relation $\vec{F} = q\vec{E}$ is still valid for this field. (vi) This field can vary with time.

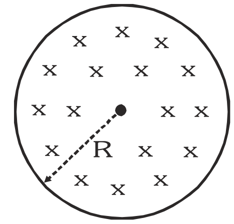
• For symmetrical situations $E\ell = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$

ℓ = the length of closed loop in which electric field is to be calculated

A = the area in which magnetic field is changing.

Direction of induced electric field is the same as the direction of included current.

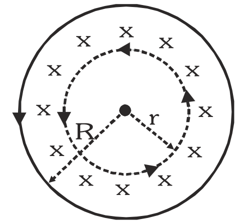
Example



The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $\alpha \frac{\text{tesla}}{\text{second}}$. Find the magnitude of electric field as a function of r, the distance from the geomatric centreof the region.

Solution

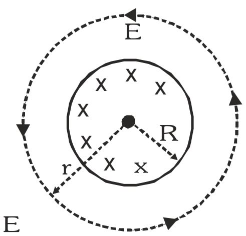
For $r \le R$:



$\because \quad E\,\ell = A\left|\dfrac{dB}{dt}\right|$

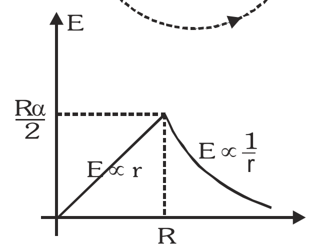
$\therefore \quad E\,(2\pi r) = (\pi r^2)\,\alpha \Rightarrow E = \dfrac{r\alpha}{2} \Rightarrow E \propto r$

E-r graph is straight line passing through origin.



At $\quad r = R, \qquad E = \dfrac{R\alpha}{2}$

For $r \ge R$:



$\because \quad E\,\ell = A\left|\dfrac{dB}{dt}\right|$

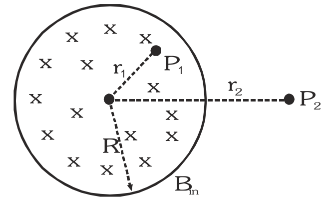
$\therefore \quad E\,(2\pi r) = (\pi R^2)\,\alpha \Rightarrow E = \dfrac{\alpha R^2}{2r} \Rightarrow E \propto \dfrac{1}{r}$

Example

For the situation described in figure the magnetic field changes with time according to $B = (2.00\,t^3 - 4.00\,t^2 + 0.8)$ T and $r_2 = 2R = 5.0$ cm

(a) Calculate the force on an electron located at P_2 at $t = 2.00$ s

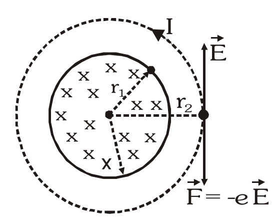
(b) What are the magnetude and direction of the electric field at P_1 when $t = 3.00$ s and $r_1 = 0.02$m.



Solution

$$E\,\ell = A\left|\frac{dB}{dt}\right| \Rightarrow E = \frac{\pi R^2}{2\pi r_2}\frac{d}{dt}(2t^3 - 4t^2 + 0.8) = \frac{R^2}{2r_2}(6t^2 - 8t)$$

(a) Force on electron at P_2 is $F = eE$



$$\therefore \text{ at } t = 2\text{ s} \quad F = \frac{1.6\times10^{-19}\times(2.5\times10^{-2})^2}{2\times5\times10^{-2}}\times[6(2)^2 - 8(2)]$$

$$= \frac{1.6}{4}\times2.5\times10^{-21}\times(24-16) = 8\times10^{-21}\,\text{N at } t = 2\text{s},$$

$\dfrac{dB}{dt}$ is positive so it is increasing.

$\therefore$ direction of induced current and E are as shown in figure and hence force of electron having charge $-e$ is right perpendicular to r_2 downwards

(b) For $r_1 = 0.02$ m and at $t = 3$s, $E = \dfrac{\pi r_1^2}{2\pi r_1}(6t^2 - 8t) = \dfrac{0.02}{2}\times[6(3)^2 - 8(3)] = 0.3$ V/m at $t = 3$sec, $\dfrac{dB}{dt}$ is positive so B is increasing and hence direction of E is same as in case (a) and it is left perpendicular to r_1 upwards.

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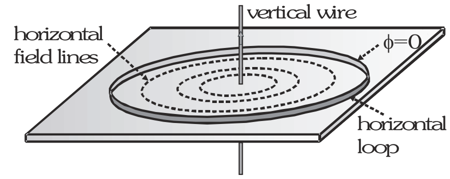
NO E.M.I. CASES

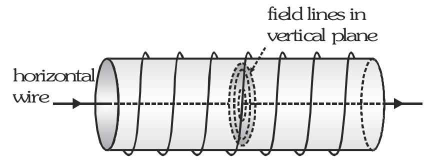
Condition of No EMI If :

No flux linkage through the coil $\phi=0$

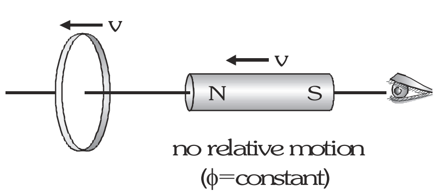
or Flux linkage through the coil $\phi=$ constant

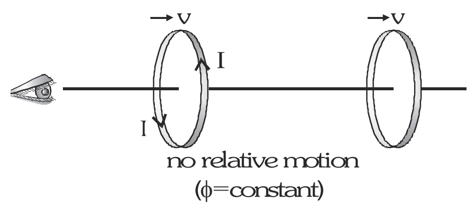


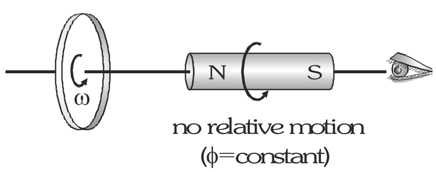
- If current I increases with respect to time no induced current in loop because no flux associated with it as plane of circular field lines of straight conductor is parallel to plane of loop.

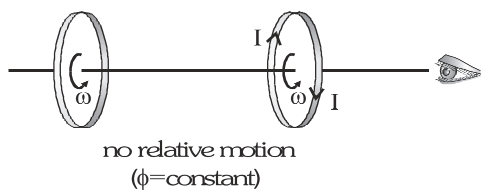


- If current I increases with respect to time no induced parameter in solenoid because no flux associated with solenoid

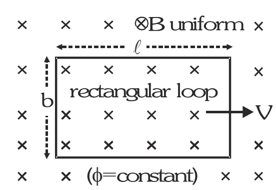
- 



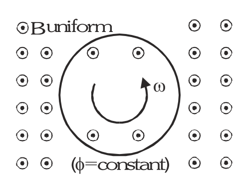




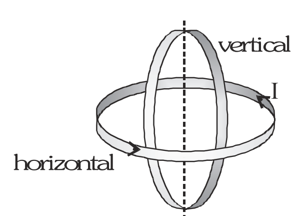
- Any rectangular coil or loop translates within the uniform transverse magnetic field its flux remains constant.



- Any coil or loop rotates about its geometrical axis in uniform transverse magnetic field its flux remains const.



- No flux associated for the coils or loops which are placed in mutually perpendicular planes. Hence If current of one either increase or decrease, there is no effect on flux of other.



METHODS OF PRODUCING INDUCED EMF (TYPES OF EMI)

Emf can be induced in a closed loop by changing the magnetic flux linked with a circuit.

The magnetic flux is $\phi = BA\cos\theta$ Magnetic flux can be changed by one of the following methods :

(i) Changing the magnetic field B. (Static emi)

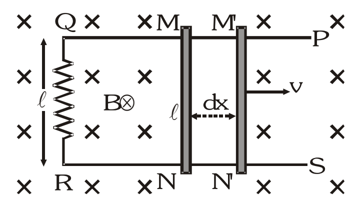
(ii) Changing the area A of the coil and (dynamic emi)

(iii) Changing the relative orientation θ of $\vec{B}$ and $\vec{A}$ (Periodic emi)

Induced emf by changing the magnetic field B

When there is a relative motion between the magnet and a closed loop, the magnetic lines of force passing through it changes, which results in change in magnetic flux. The changing magnetic flux produces induced emf in the loop.

Induced emf by changing the area of the coil



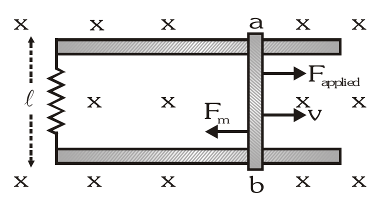
A U shaped frame of wire, PQRS is placed in a uniform magnetic field B perpendicular to the plane and vertically inward. A wire MN of length ℓ is placed on this frame. The wire MN moves with a speed v in the direction shown. After time dt the wire reaches to the position M'N' and distance covered = dx. The change in area ΔA = Length area = ℓdx

Change in the magnetic flux linked with the loop in the dt is $d\phi = B \quad \Delta A = B \quad \ell dx$

induced emf $e = \frac{d\phi}{dt} = B\ell\frac{dx}{dt} = B\ell v \because \left[v = \frac{dx}{dt}\right]$

If the resistance of circuit is R and the circuit is closed then the current through the circuit $I = \frac{e}{R} \Rightarrow I = \frac{Bv\ell}{R}$

A magnetic force acts on the conductor in opposite direction of velocity is



$$F_m = i\ell B = \frac{B^2\ell^2 v}{R}.$$

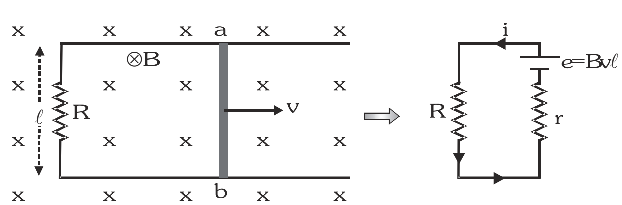
So, to move the conductor with a constant velocity v an equal and opposite force F has to be applied in the conductor. $F = F_m = \frac{B^2\ell^2 v}{R}$

The rate at which work is done by the applied force is, $P_{applied} = Fv = \frac{B^2\ell^2 v^2}{R}$

and the rate at which energy is dissipated in the circuit is, $P_{dissipated} = i^2 R = \left[\frac{Bv\ell}{R}\right]^2 R = \frac{B^2\ell^2 v^2}{R}$

This is just equal to the rate at which work is done by the applied force.

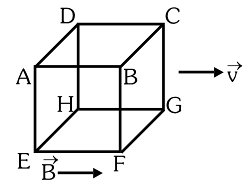
- In the figure shown, we can replaced the moving rod ab by a battery of emf $Bv\ell$ with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery.



Hence, the current in the ciruit is $i = \frac{e}{R+r} = \frac{Bv\ell}{R+r}$

Example

Twelve wires of equal lengths are connected in the form of a skeleton cube which is moving with a velocity $\vec{v}$ in the directon of magnetic field $\vec{B}$. Find the e.m.f. in each arm of cube.

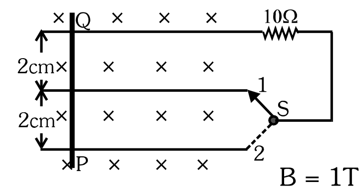


Solution

No e.m.f. is induced in any arm becuase $\vec{v}$ is parallel to $\vec{B}$ $\left(\because e_d = \vec{\ell}.(\vec{B}\times\vec{v})\right)$

Example

Wire PQ with negligible resistance slides on the three rails with 5 cm/sec. Calculate current in 10 Ω resistance when switch S is connected to (a)position 1 (b)position 2



B = 1T

Solution

(a) For position 1 Induced current $I = \dfrac{e}{R} = \dfrac{Bv\ell}{R} = \dfrac{1\times5\times10^{-2}\times2\times10^{-2}}{10}$ = 0.1 mA

(b) For position 2 Induced current $I = \dfrac{e}{R} = \dfrac{Bv(2\ell)}{R} = \dfrac{1\times5\times10^{-2}\times4\times10^{-2}}{10}$ = 0.2 mA

Example

A copper wire of length 2m placed perpendicular to the plane of magnetic field $\vec{B} = (2\hat{i}+4\hat{j})T$. If it moves with velocity $(4\hat{i}+6\hat{j}+8\hat{k})$ m/sec. Calculate dynamic emf across its ends.

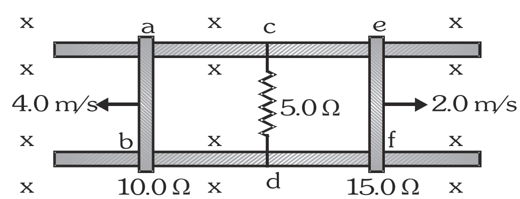
Solution

Dynamic emf $e_d = -\vec{\ell}\cdot(\vec{v}\times\vec{B}) = \vec{\ell}\cdot(\vec{B}\times\vec{v})$ $\quad (\vec{B}\times\vec{v}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 4 & 0\\ 4 & 6 & 8\end{vmatrix} = \hat{i}\,(32-0) - \hat{j}\,(16-0) + \hat{k}\,(12-16)$

$= 32\hat{i}-16\hat{j}-4\hat{k}$, $e_d = (2\hat{k})\quad(32\hat{i}-16\hat{j}-4\hat{k}) = -8$ volt

Example

Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0 Ω resistor. The circuit also contains two metal rods having resistance of 10.0Ω and 15.0 Ω along the rails (fig). The rods are pulled away from the resistor at constantspeeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.0 Ω resistor.



Solution

Two conductors are moving in uniform magnetic field, so motional emf will induced across them.

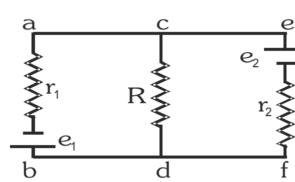
The rod ab will act as a source of emf $e_1 = Bv\ell = (0.01)(4.0)(0.1) = 4 \quad 10^{-3}$ V

and internal resistance $r_1 = 10.0\ \Omega$

Similarly, rod ef will also act as source of emf $e_2 = (0.01)(2.0)(0.1) = 2 \quad 10^{-3}$ V

and internal resistance $r_2 = 15.0\ \Omega$

From right hand rule : $V_b > V_a$ and $V_e > V_f$ Also R = 5.0 Ω,



$$E_{eq} = \frac{e_1 r_2 - e_2 r_1}{r_1 + r_2} = \frac{6\times10^{-3} - 20\times10^{-3}}{15+10} = \frac{40}{25}\times10^{-3} = 1.6\times10^{-3} \text{ volt}$$

$$r_{eq} = \frac{15\times10}{15+10} = 6\,\Omega \text{ and } I = \frac{E_{eq}}{r_{eq}+R} = \frac{1.6\times10^{-3}}{6+6} = \frac{1.6}{11}\times10^{-3} = \frac{8}{55}\times10^{-3} \text{ amp from d to c}$$

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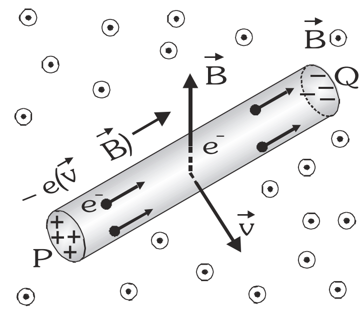
MOTIONAL EMF FROM LORENTZ FORCE



A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force, $\vec{F}_m = (\vec{v} \times \vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative chagre accumulates at Q and a positive charge at P. An electric field E is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = e\vec{E}$

The accumulation of charge at the two ends continues till these two forces balance each other.

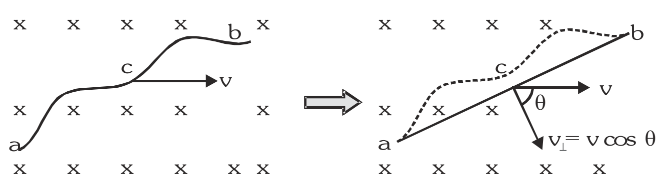
so $\vec{F}_m = -\vec{F}_e \Rightarrow \; e(\vec{v} \times \vec{B}) = -e\vec{E} \;\Rightarrow\; \vec{E} = -(\vec{v} \times \vec{B})$

The potential difference between the ends P and Q is V = $\vec{E}.\vec{\ell}$ = $(\vec{v} \times \vec{B}).\vec{\ell}$ .It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\mathcal{E}$ = B ℓ v (for $\vec{B} \perp \vec{v} \perp \vec{\ell}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $\vec{d\ell}$ of conductor the contribution de to the emf is the magnitude $d\ell$ multiplied by the component of $\vec{v} \times \vec{B}$ parallel to $\vec{d\ell}$, that is $de = (\vec{v} \times \vec{B}).\vec{d\ell}$

For any two points a and b the motional emf in the direction from b to a is,

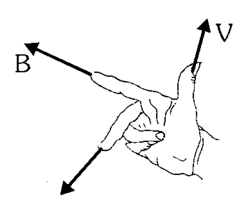


$$e = \int_b^a (\vec{v} \times \vec{B}).\vec{d\ell}$$

Motional emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab. Thus, $e_{acb} = e_{ab}$ = (length of ab) $(v_\perp)$ (B), $v_\perp$ = the component of velocity perpendicular to both $\vec{B}$ and ab. From right hand rule : b is at higher potential and a at lower potential. Hence, $V_{ba} = V_b - V_a$ = (ab) (v cosθ) (B)

Direction of induced current or HP end of the rod find out with the help of

Fleming right hand rule

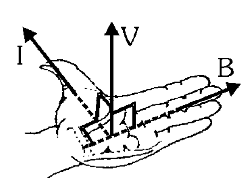


Fore finger → In external field $\vec{B}$ direction.

Thumb → In the direction of motion ($\vec{v}$) of conductor.

Middle finger→ It indicates HP end of conductor/direction of induced current.

Left hand palm rule



Fingers → In external field ($\vec{B}$) direction.

Palm → In direction of motion ($\vec{v}$) of conductor.

Thumb → It indicates HP end of conductor/direction of induced current in conductor.

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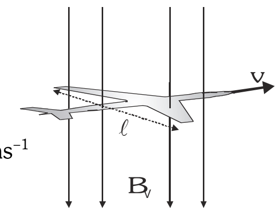
Example

An aircraft with a wing span of 40 m flies with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetic field is $1.75 \quad 10^{-5}$ T. Find the emf that develops between the tips of the wings.

Solution

The metallic part between the wing-tips can be treated as a single conductor cutting flux-lines due to vertical component of earth's magnetic field. So emf is induced between the tips of its wings.



Here l = 40 m, $B_V = 1.75 \quad 10^{-5}$ T, $v = 1080\,kmh^{-1} = \frac{1080 \times 1000}{3600} ms^{-1}$ = 300 ms^{-1}

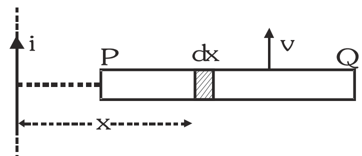
$\therefore \quad \mathcal{E} = B_V\, \ell\, v = 1.75 \quad 10^{-5} \quad 40 \quad 300 = 0.21$ V

Example

A rod PQ of length L moves with a uniform velocity v parallel to a long straight wire carrying a current i, the end P remaining at a distance r from the wire. Calculate the emf induced across the rod. Take v = 5.0 m/s, i = 100 amp, r = 1.0 cm and L = 19 cm.

Solution

The rod PQ is moving in the magnetic field produced by the current-carrying long wire. The field is not uniform throughout the length of the rod (changing with distance). Let us consider a small element of length dx at distance x from wire. if magnetic field at the position of dx is B then emf induced



$$d\mathcal{E} = B\, v\, dx = \frac{\mu_0}{2\pi}\frac{i}{x} v\, dx$$

$\therefore$ emf $\mathcal{E}$ is induced in the entire length of the rod PQ is $\mathcal{E} = \int d\mathcal{E} = \int_P^Q \frac{\mu_0}{2\pi}\frac{i}{x} v\, dx$

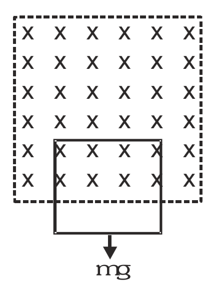
Now x = r at P, and x = r + L at Q. hence

$$\mathcal{E} = \frac{\mu_0 i v}{2\pi} = \int_r^{r+L} \frac{dx}{x} = \frac{\mu_0 i v}{2\pi}\left[\log_e x\right]_r^{r+L} = \frac{\mu_0 i v}{2\pi}[\log_e (r+L) - \log_e r] = \frac{\mu_0 i v}{2\pi} \log \frac{r+L}{r}$$

Putting the given values : $\mathcal{E} = (2 \quad 10^{-7})\,(100)\,(5.0) \log_e \frac{1.0+19}{1.0} = 10^{-4} \log_e 20$ Wb/s = $3 \quad 10^{-4}$ volt

Example

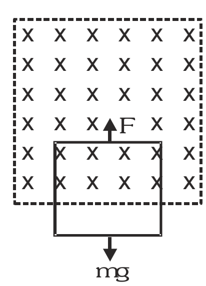
A horizontal magnetic field B is produced across a narrow gap between square iron pole-pieces as shown. A closed square wire loop of side ℓ, mass m and resistance R is allowed of fall with the top of the loop in the field. Show that the loop attains a terminal velocity given by $v = \frac{Rmg}{B^2 \ell^2}$ while it is between the poles of the magnet.



Solution

As the loop falls under gravity, the flux passing through it decreases and so an induced emf is set up in it. Then a force F which opposes its fall. When this force becomes equal to the gravity force mg, the loop attains a terminal velocity v.

The induced emf e = B v ℓ, and the induced current is $i = \frac{e}{R} = \frac{B v \ell}{R}$



The force experienced by the loop due to this current is $F = B\,\ell\, i = \frac{B^2 v \ell^2}{R}$

When v is the terminal (constant) velocity F = mg or $\frac{B^2 v \ell^2}{R} = mg$ or $v = \frac{R\,mg}{B^2 \ell^2}$

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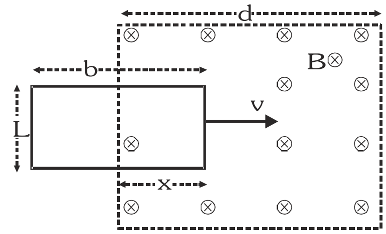
Example

Figure shows a rectangular conducting loop of resistance R, width L, and length b being pulled at constant speed v through a region of width d in which a uniform magnetic field B is set up by an electromagnet.Let L = 40 mm, b = 10 cm, d = 15 cm, R = 1.6 Ω,

B = 2.0 T and v = 1.0 m/s



(i) Plot the flux ϕ through the loop as a function of the position x of the right side of the loop.

(ii) Plot the induced emf as a function of the positioin of the loop.

(iii) Plot the rate of production of thermal energy in the loop as a function of the position of the loop.

Solution

(i) When the loop is not in the field :

The flux linked with the loop $\phi = 0$

When the loop is entirely in the field :

Magnitic flux linked with the loop

$\phi = B\,L\,b = 2 \quad 40 \quad 10^{-3} \quad 10^{-1} = 8$ mWb

When the loop is entering the field :

The flux linked with the loop $\phi = B\,L\,x$

When the loop is leaving the field :

The flux $\phi = B\,L\,[b - (x - d)]$

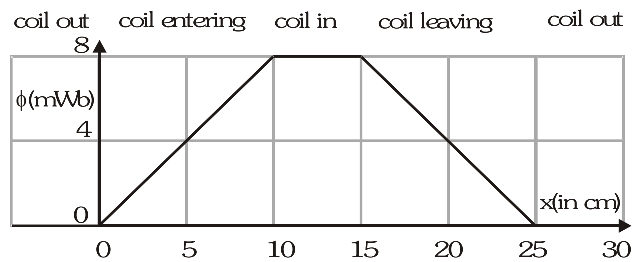


fig. (i)

(ii) Induced emf is $e = -\frac{d\phi}{dt} = -\frac{d\phi}{dx}\frac{dx}{dt} = -\frac{d\phi}{dx}v$

= – slope of the curve of figure (i)   v

The emf for 0 to 10 cm :

$$e = -\frac{(8-0)\times 10^{-3}}{(10-0)\times 10^{-2}}\times 1 = -80\,\text{mV}$$

The emf for 10 to 15 cm : e = 0   1 = 0

The emf for 15 to 25 cm :

$$e = -\frac{(0-8)\times 10^{-3}}{(25-15)\times 10^{-2}}\times 1 = +80\,\text{mV}$$

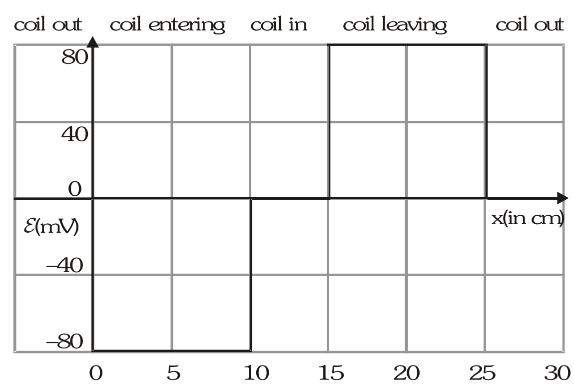


fig. (ii)

(iii) The rate of thermal energy production is $P = \frac{e^2}{R}$

for 0 to 10 cm : $P = \frac{(80\times 10^{-3})^2}{1.6} = 4\,\text{mW}$

for 10 to 15 cm : P = 0

for 15 to 25 cm : $P = \frac{(80\times 10^{-3})^2}{1.6} = 4\,\text{mW}$

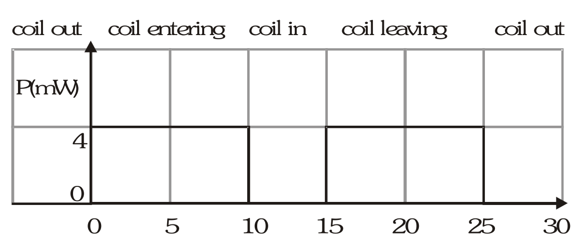


fig. (iii)

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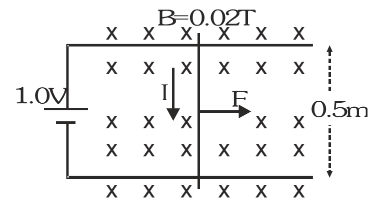
Example

Two long parallel wires of zero resistance are connected to each other by a battery of 1.0 V. The separation between the wires is 0.5 m. A metallic bar, which is perpendicular to the wires and of resistance 10 Ω, moves on these wires. When a magneticfield of 0.02 testa is acting perpendicular to the plane containing the bar and the wires. Find the steady-state veclocity of the bar. If the mass of the bar is 0.002 kg then find its velocity as a function of time.

Solution



The current in the 10Ω bar is $I = \dfrac{1.0\,V}{10\,\Omega} = 0.1\,A$

The current carrying bar is placed in the magnetic field $\vec{B}$ (0.2 T) perpendicular to the plane of paper and directed downwards.

The magnetic force of the bar is F = B I ℓ = 0.02   0.5   0.1 0 = 1 10^{-3} N

The moving bar cuts the lines of force of $\vec{B}$. If v be the instantaneous velocity of the bar, then the emf induced in the bar is $\mathcal{E}$ = Bℓv = 0.02   0.5   v = 0.01 v volt. By Lenz's law, $\mathcal{E}$ will oppose the motion of the bar which will ultimately attain a steady velocity. In this state, the induced emf $\mathcal{E}$ will be equal to the applied emf (1.0 volt).

$$\therefore\ 0.01\,v = 1.0 \quad \text{or} \quad v = \frac{1.0}{0.01} = 100\ \text{ms}^{-1}$$

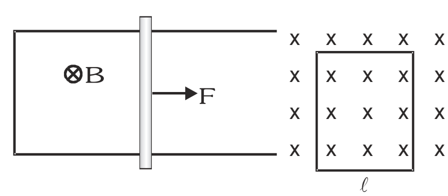
Again, a magnetic force F acts on the bar. If m be the mass of the bar, the acceleration of the rod is

$$\frac{dv}{dt} = \frac{F}{m} \Rightarrow dv = \frac{F}{m}.dt \quad \text{Integrating,} \quad \int dv = \int \frac{F}{m}dt \Rightarrow v = \frac{F}{m}t + C \ \text{(constant)}$$

If at t = 0, v = 0 then C = 0. $\therefore\ v = \dfrac{F}{m}t$ But F = 1 10^{-3} N, m = 0.002 kg $\therefore\ v = \dfrac{1\times10^{-3}}{0.002}t$ = 0.5 t

Example

In figure, a rod closing the circuit moves along a U-shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicualr to its plane. Calculate F if the rate generation of heat is P.



Solution

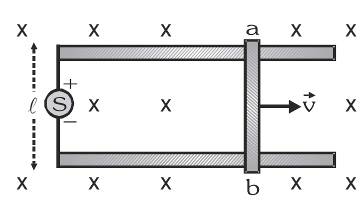
The emf induced across the ends of the rod, $\mathcal{E}$ = Bℓv

Current in the circuit, $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$ Magnetic force on the conductor, F' = IℓB, towards left

$\because$ acceleration is zero F' = F $\Rightarrow$ BIℓ = F or $I = \dfrac{F}{B\ell}$ $\because$ P = $\mathcal{E}$ I = B ℓ v $\dfrac{F}{B\ell}$ = Fv $\therefore F = \dfrac{P}{v}$

Example

The diagram shows a wire ab of length ℓ and resistance R sliding on a smooth pair of rails with a velocity v towards right. A uniform magnetic field of induction B acts normal to the plane containing the rails and the wire inwards. S is a current source providing a constant I in the circuit.



Determine the potential difference between a and b.

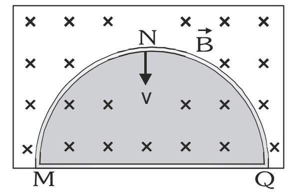
Solution

The wire ab which is moving with a velocity v is equivalent to an emf source of value B v ℓ with its positive terminal towards a.

$\therefore$ Potential difference $V_a - V_b = Bv\ell - IR$

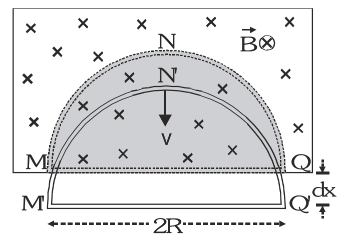
Example

A thin semicircular conduting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ (fig.). At the position MNQ, the speed of the ring is v. What is the potential difference developed across the ring at the position MNQ ?



Solution

Let semiconductor ring falls through an infinitesimally small distance dx from its initial position MNQ to M'Q'N' in time dt (fig).



decrease in area of the ring inside the magnetic field,

$dA = -\ MQQ'M' = -\ M'Q'\quad QQ' = -2R\ dx$

$\therefore$ change in magnetic flux linked with the ring,

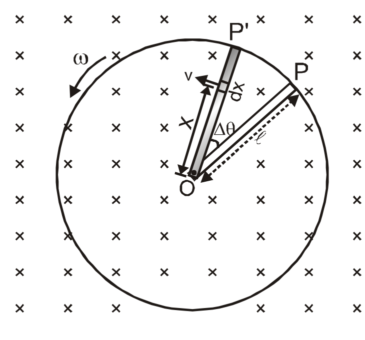
$d\phi = B\quad dA = B\quad (-R\ dx) = -\ 2BR\ dx$

The potential difference developed across the ring, $e = -\dfrac{d\phi}{dt} = -\left[-2BR\dfrac{dx}{dt}\right] = 2BRv$

the speed with which the ring is falling $v = \dfrac{dx}{dt}$

INDUCED E.M.F. DUE TO ROTATION OF A CONDUCTOR ROD IN A UNIFORM MAGNETIC FIELD

Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.



Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

So, according to Lorent's formula induced e.m.f. across this small element

$d\varepsilon = B\ v.\ dx$

$\because$ This small element dx is at distance x from O (axis of rotation)

$\therefore$ Linear velocity of this element dx is $v = \omega x$

substitute of value of v in eq^n (i) $d\varepsilon = B\ \omega\ x\ dx$

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

So, net induced e.m.f. across conducting rod $\varepsilon = \int d\varepsilon = \int_0^{\ell} B\omega\ x\ dx = \omega\ B\left(\dfrac{x^2}{2}\right)_0^{\ell}$

or $\varepsilon = \dfrac{1}{2}B\ \omega\ \ell^2$ $\quad \varepsilon = \dfrac{1}{2}B \times 2\ \pi\ f \times \ell^2$ [f = frequency of rotation]

$= B\ f\ (\pi\ \ell^2)$ $\quad$ area traversed by the rod $A = \pi\ \ell^2$ $\quad$ or $\quad \varepsilon = B\ A\ f$

Example

A wheel with 10 metallic spokes each 0.5 m long is rotated with angular speed of 120 revolutions per minute in a plane normal to the earth's mangetic field. If the earth's magnetic field at the given place is 0.4 gauss, find the e.m.f. induced between the axle and the rim of the wheel.

Solution

$\omega = 2\pi n = 2\pi \times \dfrac{120}{60} = 4\pi$, B = 0.4 G = 4 10^{-5} T, length of each spoke = 0.5 m

induced emf $e = \dfrac{1}{2} B \omega \ell^2 = \dfrac{1}{2} \times 4 \times 10^{-5} \times 4\pi \times (0.5)^2 = 6.28 \times 10^{-5}$ volt

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

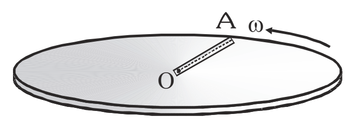
Example

A horizontal copper disc of diameter 20 cm, makes 10 revolutions/sec about a vertical axis passing through its centre. A uniform magnetic field of 100 gauss acts perpendicular to the plane of the disc. Calculate the potential difference its centre and rim in volts.

Solution

B = 100 gauss = 100 10^{-4} Wb/m^2 = 10^{-2}, r = 10 cm = 0.10 m, frequency of rotaion = 10 rot/sec

The emf induced between centre and rim $\mathcal{E} = \dfrac{1}{2} B \omega \ell^2 = \dfrac{1}{2} B \omega r^2$ ($\because$ r = ℓ)



$\omega = 2\pi f = 2 \quad 3.14 \quad 10 = 62.8\ s^{-1}$ $\therefore$ $\mathcal{E} = \dfrac{1}{2} \times 10 \times 62.8 \times (0.1)^2 = 3.14 \quad 10^{-3}$ V = 3.14 mV.

Example

A circular copper disc 10 cm in radius rotates at 20 π rad s^{-1} about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc. (a) Calculate the potential difference developed between the axis of the disc and the rim. (b) What is the induced current, if the resistance of 2Ω is connected in between axis and rim of the disc.

Solution

Here B = 0.2 T radius of the circular disc, r = 10 cm = 0.1 m resistance of the disc, R = 2Ω

angular speed of rotation of the disc, ω = 20 π rad s^{-1}

(a) If e is the induced e.m.f. produced between the axis of the disc and its rim, then

$$e = \frac{1}{2} B \omega r^2 = \frac{1}{2} \times 0.2 \times 20\pi \times (0.1)^2 = 0.0628 \text{ V}$$

(b) Induced current $I = \dfrac{e}{R} = \dfrac{0.0628}{2} = 0.0314\,A$

SELF INDUCTION

When the current through the coil changes, the magnetic flux linked with the coil also changes. Due to this change of flux a current induced in the coil itself according to lenz concept it opposes the change in magnetic flux. This phenomenon is called self induction and a factor by virtue of coil shows opposition for change in magnetic flux called cofficient of self inductance of coil.

Considering this coil circuit in two cases.

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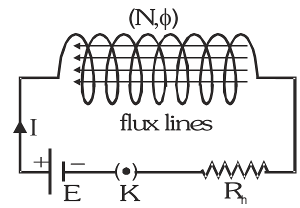
Case - I Current through the coil is constant



If $I \rightarrow B \rightarrow \phi$ (constant) $\Rightarrow$ No EMI

total flux of coil ($N\phi$) $\propto$ current through the coil

$N\phi \propto I \Rightarrow N\phi = LI \qquad L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$

where L = coefficient of self inductance of coil

S I unit of L : $1\,\frac{Wb}{amp} = 1$ Henry $= 1\,\frac{N.m}{A^2} = 1\,\frac{J}{A^2}$ **Dimensions :** $[M^1L^2T^{-2}\,A^{-2}]$

Note : L is constant of coil it **does not depends on current** flow through the coil.

Case - II Current through the coil changes w.r.t. time

If $\frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ Static EMI $\Rightarrow N\phi = LI$

$-N\frac{d\phi}{dt} = -L\frac{dI}{dt}$, $(-N\frac{d\phi}{dt})$ called total self induced emf of coil 'e_S'

$e_s = -L\frac{dI}{dt}$ **S.I. unit of L** $\rightarrow \frac{V.s}{A}$

SELF-INDUCTANCE OF A PLANE COIL

Total magnetic flux linked with N turns,

$$\phi = NBA = N\left(\frac{\mu_0 NI}{2r}\right)A = \frac{\mu_0 N^2 I}{2r}\ A = \frac{\mu_0 N^2 I}{2r}\quad \pi r = \frac{\mu_0 \pi N^2 r}{2}I \qquad \text{But } \phi = LI \therefore L = \frac{\mu_0 \pi N^2 r}{2}$$

Example

A soft iron core is introduced in an inductor. What is the effect on the self-inductance of the inductor?

Solution

Since soft iron has a large relative permeability therefore the magnetic flux and consequently the self-inductance is considerably increased.

SELF-INDUCTANCE OF A SOLENOID

Let cross-sectional area of solenoid=A, Current flowing through it=I

Length of the solenoid $=\ell$, then $\phi = NBA = N\frac{\mu_0 NI}{\ell}A = \frac{\mu_0 N^2 A}{\ell}I$

But $\phi = LI \therefore L = \frac{\mu_0 N^2 A}{\ell}$ or $L_m = \frac{\mu_0 \mu_r N^2 A}{\ell}$

If no iron or similar material is nearby, then the value of self-inductance depends only on the geometrical factors (length, cross-sectional area, number of turns).

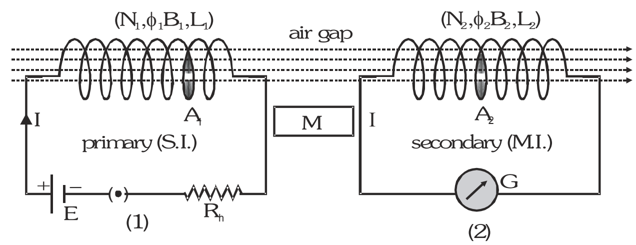
Example

The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at the rate of 0.8 As^{-1}. Find the emf induced in it.

Solution

$$|\mathcal{E}| = L\frac{dI}{dt} = \frac{\mu_0 N^2 A}{\ell}\cdot\frac{dI}{dt} = \frac{4\pi\times10^{-7}\times(240)^2\times\pi\times(0.02)^2}{0.12}\times 0.8 = 6\times10^{-4}\,V$$

MUTUAL INDUCTION



Whenever the current passing through primary coil or circuit change then magnetic flux neighbouring secondary coil or circuit will also change. Acc. to Lenz for opposition of flux change, so an emf induced in the neighbouring coil or circuit.

This phenomenon called as 'Mutual induction'. In case of mutual inductance for two coils situated close to each other, flux linked with the secondary due to current in primary.

Due to Air gap always $\phi_2<\phi_1$ and $\phi_2 = B_1A_2$ $(\theta=0)$.

Case - I When current through primary is constant

Total flux of secondary is directly proportional to current flow through the primary coil

$$N_2\,\phi_2 \propto I_1 \Rightarrow N_2\,\phi_2 = MI_1,\; M=\frac{N_2\phi_2}{I_1}=\frac{N_2B_1A_2}{I_1}=\frac{(\phi_T)_s}{I_p}$$ where M : is coefficient of mutual induction.

Case - II When current through primary changes with respect to time

If $\frac{dI_1}{dt}\rightarrow\frac{dB_1}{dt}\rightarrow\frac{d\phi_1}{dt}\rightarrow\frac{d\phi_2}{dt}\Rightarrow$ Static EMI

$$N_2\phi_2 = MI_1 \quad -N_2\frac{d\phi_2}{dt} = -M\frac{dI_1}{dt},\;\left[-N_2\frac{d\phi}{dt}\right]$$

$$e_m = -M\left(\frac{dI_1}{dt}\right)$$
Secondary ← e_m ; $\frac{dI_1}{dt}$ → Primary

called total mutual induced emf of secondary coil e_m.

- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

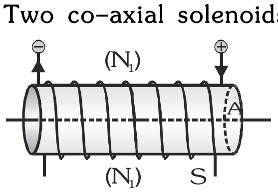
'M' depends on :

- Number of turns (N_1, N_2).
- Cofficient of self inductance (L_1, L_2).
- Area of cross section.
- Magnetic permeability of medium (μ_r).
- Distance between two coils (As $d\downarrow\; = M\uparrow$).
- Orientation between two coils.
- **Coupling factor** 'K' between two coils.

DIFFERENT COEFFICIENT OF MUTUAL INDUCTANCE

- In terms of their number of turns
- In terms of their coefficient of self inductances
- **In terms of their nos of turns (N_1 , N_2)**

(a) Two co–axial solenoids :– $(M_{s_1s_2})$



Coefficient of mutual inductance between two solenoids

$$M_{s_1s_2} = \frac{N_2B_1A}{I_1} = \frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{\ell}\right]A \Rightarrow M_{s_1s_2}=\left[\frac{\mu_0N_1N_2A}{\ell}\right]$$

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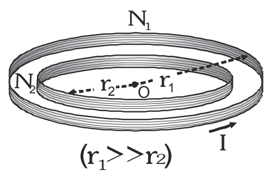
(b) Two plane concentric coils ($M_{c_1c_2}$)

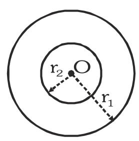


$$M_{c_1c_2} = \frac{N_2B_1A_2}{I_1} \quad \text{where } B_1 = \frac{\mu_0N_1I_1}{2r_1}, \quad A_2 = \pi r_2^2$$

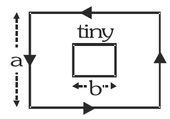
$$M_{c_1c_2} = \frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{2r_1}\right](\pi r_2^2) \Rightarrow M_{c_1c_2} = \frac{\mu_0N_1N_2\pi r_2^2}{2r_1}$$

Two concentric loop : Two concentric square loops : A square and a circular loop

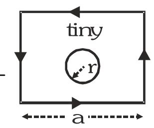
$M \propto \frac{r_2^2}{r_1}$ ($r_1 >> r_2$)



$M \propto \frac{b^2}{a}$



$M \propto \frac{r^2}{a}$

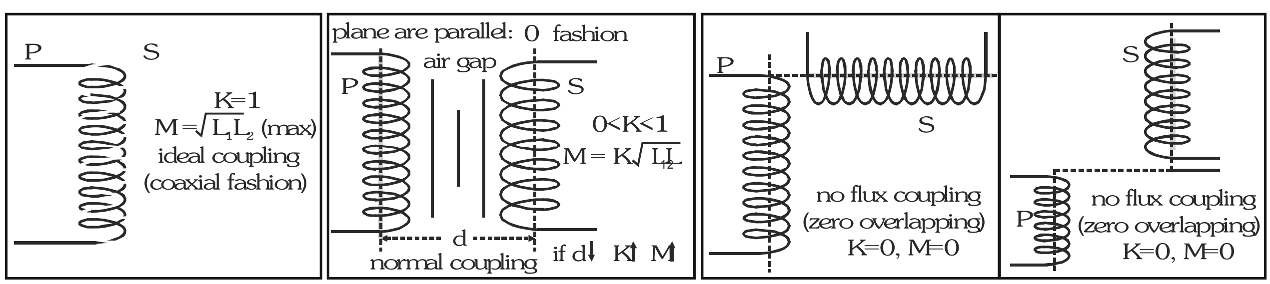


In terms of L_1 and L_2 : For two magnetically coupled coils :-

$M = K\sqrt{L_1L_2}$ here 'K' is coupling factor between two coils and its range $0 \le K \le 1$

- For ideal coupling $K_{max} = 1 \Rightarrow M_{max} = \sqrt{L_1L_2}$ (where M is geometrical mean of L_1 and L_2)
- For real coupling ($0 < K < 1$) $M = K\sqrt{L_1L_2}$
- Value of coupling factor 'K' decided from fashion of coupling.

• **Different fashion of coupling**



'K' also defined as $K = \frac{\phi_s}{\phi_p} = \frac{\text{mag. flux linked with secondary (s)}}{\text{mag. flux linked with primary (p)}}$

INDUCTANCE IN SERIES AND PARALLEL

Two coil are connected in series : Coils are lying close together (M)

If $M = 0$, $L = L_1 + L_2$ If $M \ne 0$ $L = L_1 + L_2 + 2M$

(a) When current in both is in the same direction Then $L = (L_1 + M) + (L_2 + M)$

(b) When current flow in two coils are mutually in opposite directions.

$L = L_1 + L_2 - 2M$

Two coils are connected in parallel :

(a) If $M = 0$ then $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1L_2}{L_1 + L_2}$ (b) If $M \ne 0$ then $\frac{1}{L} = \frac{1}{(L_1 + M)} + \frac{1}{(L_2 + M)}$

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Example

On a cylindrical rod two coils are wound one above the other. What is the coefficient of mutual induction if the inductance of each coil is 0.1 H ?

Solution

One coil is wound over the other and coupling is tight,so K = 1, $M = \sqrt{L_1 L_2} = \sqrt{0.1 \times 0.1} = 0.1\text{H}$

Example

How does the mutual inductance of a pair of coils change when :

(i) the distance between the coils is increased ?

(ii) the number of turns in each coil is decreased ?

(iii) a thin iron rod is placed between the two coils, other factors remaining the same ?

Justify your answer in each case .

Solution

(i) The mutual inductance of two coils, decreases when the distance between them is increased. This is because the flux passing from one coil to another decreases.

(ii) Mutual inductance $M = \dfrac{\mu_0 N_1 N_2 A}{\ell}$ i.e., $M \propto N_1 N_2$ Clearly, when the number of turns N_1 and N_2 in the two coils is decreased, the mutual inductance decreases.

(iii) When an iron rod is placed between th two coils the mutual inductance increases, because $M \propto$ permeability (μ)

Example

A coil is wound on an iron core and looped back on itself so that the core has two sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self-inductance ? Will it be larger or small ?

Solution

As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

This situation is similar to two coils connected in series and producing fluxes in opposite directions. Therefore, their equivalent inductance must be $L_{eq} = L + L - 2M = L + L - 2L = 0$

Example

A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2 10^{-3} m^2. Around its central section a coil of 300 turns is closely would. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Solution

$$M = \frac{\mu_0 N_1 N_2 A}{\ell} = \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3} = 3 \times 10^{-3}\text{H}$$

$$\mathcal{E} = -M\frac{dI}{dt} = -3 \times 10^{-3}\left[\frac{-2-2}{0.25}\right] = 48 \times 10^{-3}\text{V} = 48 \text{ mV}$$

ENERGY STORED IN INDUCTOR

The energy of a capacitor is stored in the electric field between its plates. Similary, an inductor has the capability of storing energy in its magnetic field.An increasing current in an inductor causes an emf between its terminals.

Power P = The work done per unit time $= \frac{dW}{dt} = -ei = -\left[L\frac{di}{dt}\right]i = -L\,i\frac{di}{dt}$

here i = instanteneous current and L = inductance of the coil

From dW = – dU (energy stored) so $\frac{dW}{dt} = -\frac{dU}{dt}$ $\therefore$ $\frac{dU}{dt} = Li\frac{di}{dt} \Rightarrow dU = Li\,di$

The total energy U supplied while the current increases from zero to final value i is,

$$U = L\int_0^I i\,di = \frac{1}{2}L(i^2)_0^I \quad \therefore U = \frac{1}{2}L\,I^2$$

the energy stored in the magnetic field of an inductor when a current I is $= \frac{1}{2}L\,I^2$.

The source of this energy is the external source of emf that supplies the current.

- After the current has reached its final steady state value I, $\frac{di}{dt} = 0$ and no more energy is input to the inductor.
- When the current decreases from i to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}Li^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch the current decreases very rapidly, the induced emf is very large and the energy may be dissipated in an arc the switch.

MAGNETIC ENERGY PER UNIT VOLUME OR ENERGY DENSITY

- The energy is an inductor is actually stored in the magnetic field within the coil. For a long solenoid its magnetic field can be assumed completely within the solenoid.

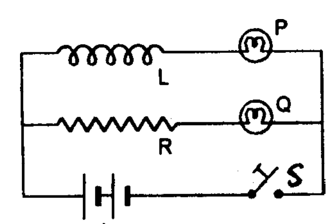
  The energy U stored in the solenoid when a current I is,

$$U = \frac{1}{2}L\,I^2 = \frac{1}{2}(\mu_0\,n^2\,V)I^2 \qquad (L = \mu_0\,n^2\,V)\ (V = \text{Volume} = A\ell)$$

The energy per unit volume $u = \frac{U}{V}$ $= \frac{1}{2}\mu_0\,n^2\,I^2 = \frac{(\mu_0\,n\,I)^2}{2\mu_0} = \frac{B^2}{2\mu_0}$ $(B = \mu_0\,n\,I)$ $\therefore$ $u = \frac{1}{2}\frac{B^2}{\mu_0}$

Example

Figure shows an inductor L a resistor R connected in paralled to a battery through a switch. The resistance of R is same as that of the coil that makes L. Two identical bulb are put in each arm of the circuit.



(a) Which of two bulbs lights up earlier when S is closed?

(b) Will the bulbs be equally bright after some time?

Solution

(i) When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of lamp P so lamp Q light up earlier.

(ii) Yes. At steady state inductive effect becomes meaningless so both lamps become equally bright after some time.

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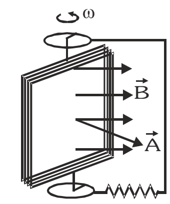
PERIODIC EMI



Let a coil intially placed perpendicular to uniform magnetic field. Now this coil starts rotation about an axis that the flux linked with the coil change due to change in oriantation of area vector $\vec{A}$ with respect to magnetic field $\vec{B}$

Angle in between area vector $\vec{A}$ and magnetic field $\vec{B}$ is θ then

Example

A coil of 160 turns of cross-sectional area 250 cm^2 rotates at an angular velocity of 300 radian/sec about an axis parallel to the plane of the coil in a uniform magnetic field of 0.6 Wb/m^2. What is the maximum emf induced in the coil ? If the coil is connected to a resistance of 2 ohm, what is the maximum torque that has to be delivered to maintain its motion.

Solution

The instantaneous induced emf is $\mathcal{E}$ = NBA ω sin ωt $\therefore$ NBA ω

$\therefore \quad \mathcal{E}_{max} = N\ B\ A\ \omega = 160 \quad 0.6 \quad (250 \quad 10^{-4}) \quad 300 = 720$ volt

The maximum current through the coil is $i_{max} = \dfrac{\mathcal{E}_{max}}{R} = \dfrac{720}{2} = 360$ amp.

The torque on a current-carrying coil placed in a magnetic field is τ = BINAsin θ = BINA sinωt

$\therefore$ maximum torque = B I N A = 0.6 360 160 (250 10^{-4}) = 864 newton meter.

By Lenz's law, this torque opposes the rotaion of the coil. Hence to maintain the rotation an equal torque must be inserted in the opposite direction. Therefore the required torque is 864 N-m.

Example

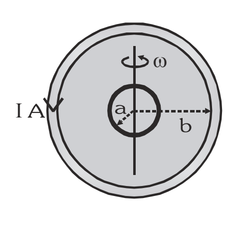
A very small circular loop of area 5 10^{-4} m^2, resistance 2 ohm and negligible inductance is initially coplanar and concentric with a much larger fixed circular loop of radius 0.1 m. A constant current of 1 ampere is passed in the bigger loop and the smaller loop is rotated with angular velocity ω rad/s about a diameter. Calculate (a) the flux linked with the smaller loop (b) induced emf and induced current in the smaller loop as a function of time.

Solution

(a) The field at the centre of larger loop $B_1 = \dfrac{\mu_0 I}{2R} = \dfrac{2\pi \times 10^{-7}}{0.1}$ = 2π 10^{-6} Wb/m^2

is initially along the normal to the area of smaller loop. Now as the smaller loop (and hence normal to its plane) is rotating at angular velocity ω, with respect to $\vec{B}$ so the flux linked with the smaller loop at time t is, $\phi_2 = B_1 A_2 \cos\theta = (2\pi \quad 10^{-6})\ (5 \quad 10^{-4}) \cos \omega t$

i.e., $\phi_2 = \pi \quad 10^{-9} \cos \omega t$ Wb



(b) The induced emf in the smaller loop $e_2 = -\dfrac{d\phi_2}{dt} = -\dfrac{d}{dt}(\pi \times 10^{-9} \cos \omega t)$

$= \pi \quad 10^{-9}\ \omega \sin \omega t$ volt

(c) The induced current in the smaller loop is, $I_2 = \dfrac{e_2}{R} = \dfrac{1}{2}\pi\omega \times 10^{-9} \sin \omega t$ ampere

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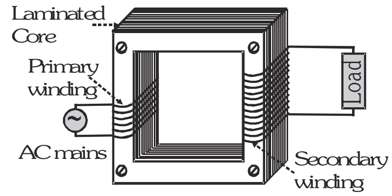
TRANSFORMER



Working principle

Mutual induction

Transformer has basic two section

(a) **Shell** : Consist of primary and secondary coil of copper.

The effective resistance between primary and secondary coil is infinite because electric circuit between two is open ($R_{ps} = \infty$)

(b) **Core** : Which is between two coil and magnetically coupled two coils. Two coils of transformer would on the same core. The alternating current passing through the primary creats a continuously changing flux through the core. This changing flux induces alternating emf in secondary.

Work

It regulates AC voltage and transfer the electrical electrical power without change in freqency of input supply. (The alternating current changes itself.)

Special Points

- It can't work with D.C. supply, and if a battery is connected to its primary, then output is across scondary is always zero ie. No working of transformer.
- It can't called 'Amplifier' as it has no power gain like **transistor**.
- It has no moving part, hence there are no mech. losses in transformer.

Types : According to voltage regulation it has two –

(i) Step up transformer : $N_S > N_P$ (ii) Step down transformer $N_S < N_P$

Step up transformer : Converts **low voltage high current** in to **High voltage low current**

Step down transformer : Converts **High voltage low current** into **low voltage high current.**

Power transmission is carried out always at "**High voltage low current**" so that voltage drop and power losses are minimum in transmission line.

voltage drop = $I_L R_L$, I_L = line current R_L = total line resistance,

$I_L = \dfrac{\text{power to be transmission}}{\text{line voltage}}$ power losses = $I_L^2 R_L$

High voltage coil having more number of turns and always **made of thin wire** and **high current coil** having less number of turns and always **made of thick wires.**

Ideal Transformer : (η = 100%)

(a) **No flux leakage** $\phi_s = \phi_p \Rightarrow \dfrac{-d\phi_s}{dt} = \dfrac{-d\phi_p}{dt}$

$e_s = e_p = e$ induced emf per turn of each coil is also same.

total induced emf for secondary $E_s = N_s e$ total induced emf for primary $E_p = N_p e$

$\dfrac{E_s}{E_p} = \dfrac{N_s}{N_p} = n$ or p where n : turn ratio , p : transformation ratio

(b) No load condition

$V_P = E_P$ and $E_S = V_S$ $\quad \frac{V_S}{V_P} = \frac{N_S}{N_P}$ from (i) and (ii) $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ = n or p

(c) No power loss

$P_{out} = P_{in}$ and $V_S I_S = V_P I_P$ $\quad \frac{V_S}{V_P} = \frac{I_P}{I_S}$ valid only for ideal transformer

from equation (iii) and (iv) $\quad \frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = n$ or p

Note : Generally transformers deals in ideal condition i.e. $P_{in} = P_{out}$, if other information are not given.

Real transformer ($\eta \neq 100\%$)

Some power is always lost due to flux leakage, hysteresis, eddy currents, and heating of coils.

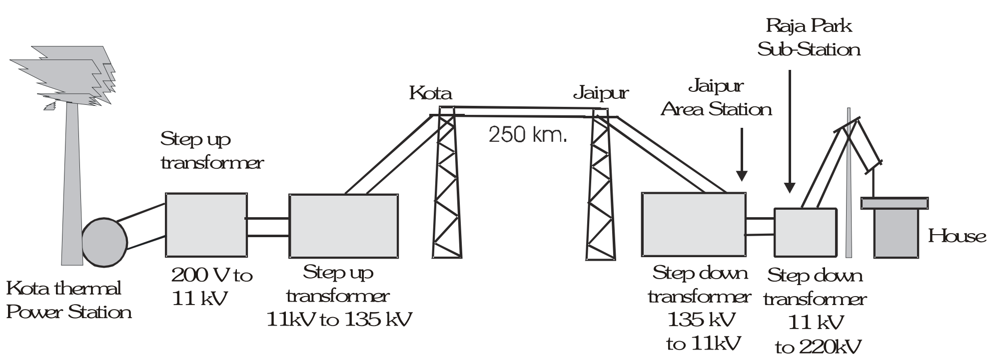
hence $P_{out} < P_{in}$ always. efficiency of transformer $\eta = \frac{P_{out}}{P_{in}} = \frac{V_S}{V_P} \cdot \frac{I_S}{I_P} \times 100$

Applications :

The most important application of a transformer is in long distance transmission of electric power from generating station to consumers hundreds of kilometers away through transmission lines at reduced loss of power.

Transmission lines having resistance R and carrying current I have loss of power = I^2R.

This loss is reduced by reducing the current by stepping up the voltage at generating station. This high voltage is transmitted through high-tension transmission lines supported on robust pylons (iron girder pillars). The voltage is stepped down at consumption station. A typical arrangement is shown below :



Example

A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric power plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets from the line through a 4000 – 220 V step down transformer at a sub-station in the town.

(b) Estimate the line power loss in the form of heat.

(b) How much power must be plant supply, assuming there is a negligible power loss due to leakage?

(c) Characterise the step up transformer at the plant.

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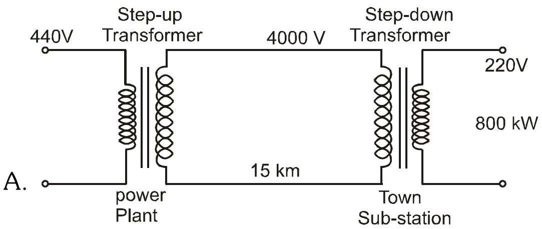
Solution

The diagram shows the network :



For sub-station, P = 800 kW = 800 10^3 watts

V = 220 V

$$I_S = \frac{P}{V} = \frac{800\times10^3}{220} = \frac{40}{11} \quad 10^3 \text{ A.}$$

Primary current (I_p) in sub-station transformer will be given by

$$4000 \quad I_P = 220 \quad I_S,\ I_P = \frac{220\times40\times10^3}{11\times4000} = 200 \text{ A}$$

(a) Hence transmission line current = 200 A

transmission line resistance = 2 15 0.5 = 15 Ω

transmission line power loss = I^2R = 200 200 15 = 6 10^5 watt = 600 kW.

(b) power to be supplied by plant = power required at substation + loss of power of transmission

= 800 + 600 = 1400 kW.

(c) Voltage in secondary at power plant has characteristics $= \frac{\text{Power}}{\text{Current}} = \frac{1400 \text{ kW}}{200 \text{ A}} = \frac{1400 \times 1000}{200} = 7000 \text{ V}$

Step-up transformer at power plant has characteristics 440 - 7000 V.

Example

A power transmission line feeds input power at 2300 V to a step down transformer having 4000 turns in its primary. What should be the number of turns in the secondary to get output power at 230 V?

Solution

E_p = 2300 V ; N_P = 4000, E_S = 230 V $\frac{E_S}{E_P} = \frac{N_S}{N_P} \therefore N_S = N_P \times \frac{E_S}{E_P} = 4000 \times \frac{230}{2300} = 400$

Example

The output voltage of an ideal transformer, connected to a 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24V, 24W calculate the current in the primary coil of the circuit.

Solution

E_P = 240 V, E_S = 24 V, $E_S I_S$ = 24 W Current in primary coil $I_P = \frac{E_S I_S}{E_P} = \frac{24}{240} = 0.1\text{A}$

LOSSES OF TRANSFORMER

(a) Copper or joule heating losses

Where : There losses occurs in both coils of shell part

Reason : Due to heating effect of current ($H = I^2Rt$)

Remmady : To minimise these losses, high current coil always made up with thick wire and for removal of produced heat circulation of mineral oil should be used.

(b) Flux leakage losses

Where : There losses occurs in between both the coil of shell part.

Cause : Due to air gap between both the coils.

Remmady : To minimise there losses both coils are tightly wound over a common soft iron core (high magnetic permeability) so a closed path of magnetic field lines formed itself within the core and tries to makes coupling factor K → 1

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(c) **Iron losses**

Where : There losses occurs in core part.

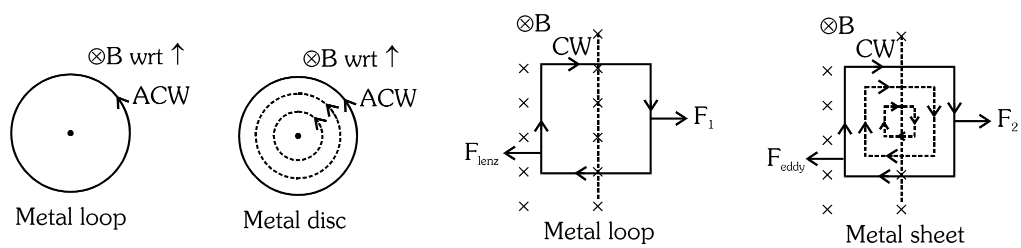
Types : (i) Hysteresis losses (ii) Eddy currents losses

(i) **Hysteresis losses**

Cause : Transformer core always present in the effect of alternating magnetic field ($B = B_0 \sin\omega t$) so it will magnetised & demagnetised with very high frequeny ($f = 50$ Hz).During its demagnetization a part of magnetic energy left inside core part in form of residual magnetic field. Finally this residual energy waste as heat.

Remmady : To minimise these losses material of transformer core should be such that it can be easily magnetised & demagnetised. For this purpose soft ferromagnetic material should be used. For example soft iron (low retentivity and low coercivity)

EDDY CURRENTS (or Focalt's currents)



($F_1 <<< F_2$)

- Eddy currents are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes.
- Eddy currents tend to follow the path of least resistance inside a conductor. So they from irregularly shaped loops. However, their directions are not random, but guided by Lenz's law.
- Eddy currents have both undesirable effects and practically useful applications.

Applications of eddy currents :

(i)	Induction furnace	(ii)	Electromagnetic damping
(iii)	Electric brakes	(iv)	Speedometers
(v)	Induction motor	(vi)	Electromagnetic shielding
(vii)	Inductothermy	(viii)	Energy meters

GOLDEN KEY POINTS

- These currents are produced only in closed path within the entire volume and on the surface of metal body. Therefore their measurement is impossible.
- Circulation plane of these currents is always perpendicular to the external field direction.
- Generally resistance of metal bodies is low so magnitude of these currents is very high.
- These currents heat up the metal body and some time body will melt out (Application : Induction furnace)
- Due to these induced currents a strong eddy force (or torque) acts on metal body which always apposes the translatory (or rotatory) motion of metal body, according to lenz.
- **Transformer**

  Cause : Transformer core is always present in the effect of alternating magnetic field ($B = B_0 \sin\omega t$). Due to this eddy currents are produced in its volume, so a part of magnetic energy of core is wasted as heat.

  Remmady : To minimise these losses transformer core should be laminated. with the help of lamination process, circulation path of eddy current is greatly reduced & net resistance of system is greatly increased. So these currents become

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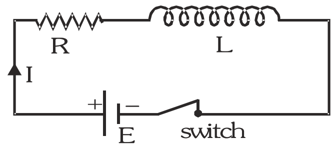
R-L DC CIRCUIT

Current Growth



(i) **emf equation** $E = IR + L\frac{dI}{dt}$

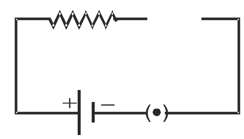
(ii) **Current at any instant**

When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant 't' given by $I = I_0\left[1 - e^{\frac{-t}{\lambda}}\right]$

t = 0 (just after the closing of key) $\Rightarrow I = 0$

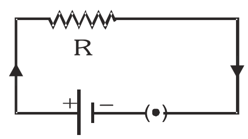
$t = \infty$ (some time after closing of key) $\Rightarrow I \to I_0$

(iii) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero.



Open circuit, t = 0, I = 0   Inductor provide infinite resistence

(iv) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and current in circuit is constant.



Short circuit, $t \to \infty$, $I \to I_0$, Inductor provide zero resistence $I_0 = \frac{E}{R}$

(Final, steady, maximum or peak value of current) or ultimate current

Note : Peak value of current in circuit does not depends on self inductance of coil.

(v) **Time constant of circuit (λ)**

$\lambda = \frac{L}{R}$ sec. It is a time in which current increases up to 63% or 0.63 times of peak current value.

(vi) **Half life (T)**

It is a time in which current increases upto 50% or 0.50 times of peak current value.

$I = I_0(1 - e^{-t/\lambda})$, t = T, $I = \frac{I_0}{2} \Rightarrow \frac{I_0}{2} = I_0(1 - e^{-T/\lambda}) \Rightarrow e^{-T/\lambda} = \frac{1}{2} \Rightarrow e^{T/\lambda} = 2$

$\frac{T}{\lambda}\log_e e = \log_e 2 \Rightarrow T = 0.693\lambda \Rightarrow T = 0.693\frac{L}{R}$ sec.

(vii) **Rate of growth of current at any instant :–**

$\left[\frac{dI}{dt}\right] = \frac{E}{L}(e^{-t/\lambda}) \quad \Rightarrow \quad t = 0 \Rightarrow \left[\frac{dI}{dt}\right]_{max} = \frac{E}{L} \quad t = \infty \Rightarrow \left[\frac{dI}{dt}\right] \to 0$

Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.

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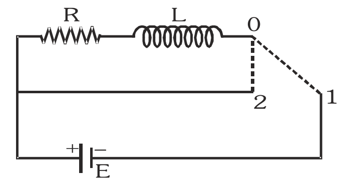
Current Decay



(i) Emf equation $IR + L\dfrac{dI}{dt} = 0$

(ii) Current at any instant

Once current acquires its final max steady value, if suddenly switch is put off then current start decreasing exponentially wrt to time. At switch put off condition t = 0, $I = I_0$, source emf E is cut off from circuit $I = I_0(e^{-t/\lambda})$

Just after opening of key $t = 0 \quad \Rightarrow I = I_0 = \dfrac{E}{R}$

Some time after opening of key $t \to \infty \quad \Rightarrow I \to 0$

(iii) Time constant (λ)

It is a time in which current decreases up to 37% or 0.37 times of peak current value.

(iv) Half life (T)

It is a time in which current decreases upto 50% or 0.50 times of peak current value.

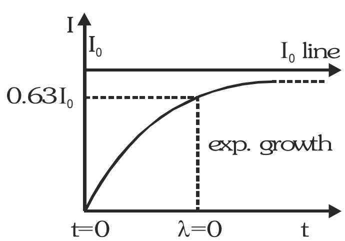
(v) Rate of decay of current at any instant

$$\left[-\frac{dI}{dt}\right] = \left[\frac{E}{L}\right]e^{-t/\lambda} \quad t = 0 \;,\; \left[-\frac{dI}{dt}\right]_{max.} = \frac{E}{L} \quad t \to \infty \Rightarrow \quad \left[-\frac{dI}{dt}\right] \to 0$$

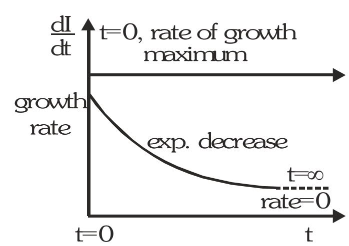
Graph for R–L circuit :–

Current Growth :–

(a)

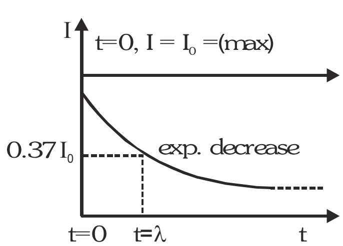


(b)

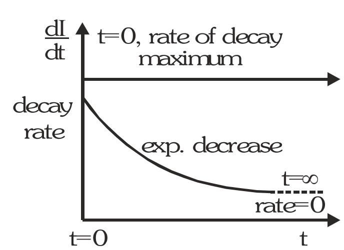


Current decay :–

(a)



(b)



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